EFFECT OF A MODULATED MAGNETIC FIELD ON THE STABILITY OF A NONISOTHERMAL MAGNETIZABLE FLUID

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The effect of variable and rotating magnetic fields on the thermal convective stability of a magnetizable fluid is investigated. The nature of the dependence of the stability boundary on the magnitudes of the magnetic field and the modulation amplitude and frequency is determined.

Until now, mainly cases of inhomogeneous external fields [1-3] have been considered in investigations of the convective stability of a magnetizable fluid. In these situations the mechanism of thermomagnetic convection is explained and determined by the magnetic field gradient, but the description is based on the gravitational analogy.

Within the framework of this standard approach, a homogeneous magnetic field should not affect the convective stability of a magnetizable fluid. The specifics of the effect of homogeneous magnetic fields on thermoconvective processes can be explained only in taking account of the magnetic field distortions caused by the thermal perturbations.

This circumstance was first noted in [4] and was later studied in [5-7]. The convective stability of a magnetizable fluid heated from below was investigated in these papers in stationary slightly inhomogeneous and homogeneous magnetic fields. Moreover, the effect of nonstationary high-frequency magnetic fields on the thermoconvective stability was investigated in [7]. Investigations performed showed that the magnetic field perturbations caused by thermal perturbations in a nonisothermal magnetizable fluid stabilize the critical motion with wave vector parallel to the equilibrium field. If no other critical motions exist, then the magnetic field perturbations will depend on the modulation amplitude (the greater the modulation amplitude, the more stable the layer). It has been explained that the stabilization caused by the field perturbations can be full for a definite magnetic field magnitude.

It should be expected that the convective stability of a magnetizable fluid will have specific features in time-varying magnetic fields at moderate frequencies; parametric domains will probably appear in addition to the fundamental stability (instability) domains.

In this connection, the convective stability of a magnetizable fluid is investigated below in rotating and time-varying magnetic fields of arbitrary frequency.

We shall consider the magnetization of the fluid to be described by the linear equation of the "magnetic state":

$$\mathbf{M} = \chi\mathbf{H},\ \chi = \chi^* - \frac{d\chi}{dT}(T - T^*).$$

Then the velocity, temperature, and magnetic field perturbations developed in the magnetizable fluid in a homogeneous magnetic field will satisfy a system of dimensionless equations [7] (the z axis of the Cartesian coordinate system is vertical, the x and y axes are horizontal, and the following are the scales selected: layer width l for the coordinates, l^2/ν for the time, $\varkappa/l$ the velocity; γl the temperature perturbations; $\gamma H^* l(\partial\chi/\partial T)/(1+\chi)$ for the magnetic field perturbations, H^* is the equilibrium magnetic field)

$$\frac{\partial}{\partial t}\Delta v_z = \Delta\Delta v_z + \mathrm{Ra}\,\Delta_1\theta - \mathrm{D}\Delta_1\mathbf{H}_0\nabla\Phi,$$

$$\mathrm{Pr}\frac{\partial\theta'}{\partial t} = v_z + \Delta\theta,\ \Delta\Phi = \mathbf{H}_0\nabla\theta'. \tag{1}$$

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1. In order to clarify the nature of the influence of a variable homogeneous magnetic field on the thermoconvective stability of a magnetizable fluid, let us examine the stability of a plane vertical layer of the same fluid heated from below. The time-modulated homogeneous fluid is transverse to the layer and directed along the x axis: $H_0 = [1 + \delta\varphi(t)]$, where $\varphi(t)$ is the modulating function. (Such a field is a solution of the Maxwell equations and satisfies the boundary conditions if the layer is surrounded by the bulk of a solid ferromagnet with the same magnetic characteristics as the fluid.)

Let us consider the plane critical motions parallel to the channel axis:

$$v_z,\ \theta,\ \Phi \sim f(x,\ t). \tag{2}$$

Such perturbations satisfy the boundary conditions

$$v_z = \theta = \frac{d\Phi}{dx} = 0 \quad \text{for} \quad x = \pm 1/2. \tag{3}$$

Problem (1)-(3) has the solution

$$v_z,\ \theta,\ \Phi \sim f(t) \cos \pi z, \tag{4}$$

corresponding to the fundamental instability level.

2. The singularities of the effect of a rotating magnetic field on the convective stability of a magnetizable fluid are investigated in the example of a horizontal layer of such a fluid heated from below (the magnetic field is rotated in the plane of the layer). In this case the magnetic field is constant in magnitude and has the components

$$H_{x0} = \cos \omega t,\ H_{y0} = \sin \omega t,\ H_{z0} = 0.$$

This situation is investigated most simply in the free boundaries case when the boundary conditions have the form

$$v_z = \frac{d^2 v_z}{dz^2} = \theta = \Phi = 0 \quad \text{for} \quad z = \pm 1/2.$$

The condition $\Phi = 0$ corresponds to the fact that the magnetic permeability of the magnetizable fluid is much greater than the magnetic permeability of the surrounding bulk. In this case system (1) has the solution

$$v_z,\ \theta,\ \Phi \sim f(t) \exp(i\mathbf{kr}) \cos n\pi z \tag{5}$$

3. Having substituted (4) or (5) in the governing system of equations (1), after simple manipulations, we obtain an equation of Hill type

$$\ddot{\theta} + 2\varepsilon\dot{\theta} + \theta\,\{1 - \mathrm{Ra}\, b + \mathrm{D}\, b\Psi(\tau)\} = 0, \tag{6}$$

where $\varepsilon = (1 + \Pr)\sqrt{\Pr}$ and the dot denotes the derivative with respect to the new dimensionless time τ. The stability boundary can be found from an analysis of (6) for both the horizontal layer of the magnetizable fluid placed in a longitudinal rotating field, and for the vertical layer in a transverse homogeneous variable field.

In the case of the vertical layer

$$b = 1/\pi^4,\ \tau = t\sqrt{\Pr}/\pi^2,\ \Psi(\tau) = [1 + \delta\varphi(\tau)]^2.$$

For the horizontal layer

$$b = k^2/(k^2 + n^2\pi^2)^3,\ \tau = t\sqrt{\Pr}/(k^2 + n^2\pi^2),$$

$$\Psi(\tau) = [k_x \cos \omega t + k_y \sin \omega t]^2/(k^2 + n^2\pi^2).$$

The problem is to seek the stability and instability domains of the solutions of (6) as a function of the values of the governing parameters.

4. Let us first consider the stability of the vertical layer. The sinusoidal modulation $\varphi(\tau) = \sin\omega\tau$ is of greatest interest. In this case the stability domain boundaries are found analytically for the high frequencies [7], and as has been mentioned, depend monotonically on the modulation amplitude. The stability boundaries of the solutions of (6) can be found quite simply for arbitrary values of ω if the sinusoidal modulation is replaced by rectangular modulation. As is known [8, 9], the general properties of solutions of the Hill equations hardly vary under such a replacement.

Let us assume the modulation to occur according to a rectangular law (for $0 < t < \pi/\omega$ $\varphi = 1$ and for $-\pi/\omega < t < 0$ $\varphi = -1$). Then the solutions of (6) have the form:

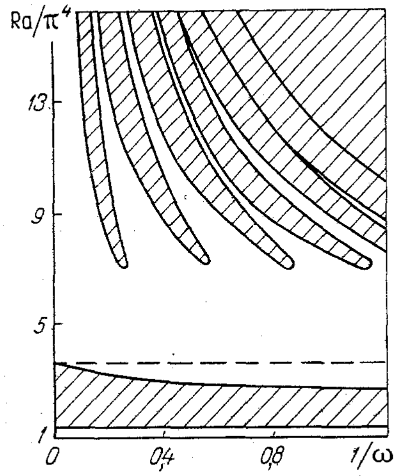


Fig. 1

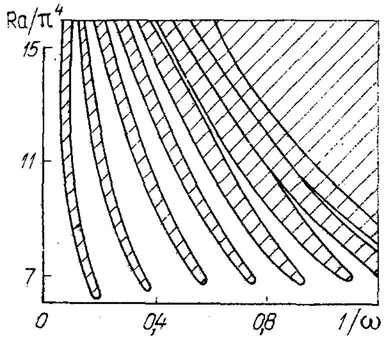


Fig. 2

Fig. 1. Dependences Ra(1/ω) for $\varepsilon = \sqrt{2}$, $\delta = 1$, $N = 1/10\pi^4$.
Fig. 2. Dependences Ra(1/ω) for $\varepsilon = \sqrt{2}$, $\delta = 1$, $N = 3/16\pi^4$.

in domain 1 $(0 < t < \pi/\omega)$

$$\theta^{(1)} = e^{-\varepsilon\tau}(c_1 \sin \alpha\tau + c_2 \cos \alpha\tau),\quad \alpha = \sqrt{1 - \mathrm{Ra}/\pi^4 + D(1+\delta)^2/\pi^4 - \varepsilon^2}, \tag{7}$$

in domain 2 $(-\pi/\omega < t < 0)$

$$\theta^{(2)} = e^{-\varepsilon\tau}(c_3 \sin \beta\tau + c_4 \cos \beta\tau),\quad \beta = \sqrt{1 - \mathrm{Ra}/\pi^4 + D(1-\delta)^2/\pi^4 - \varepsilon^2}, \tag{8}$$

θ and $\dot{\theta}$ should be continuous at $t = 0$:

$$\theta^{(2)}(0) = \pm\theta^{(1)}(0),\ \dot{\theta}^{(2)}(0) = \pm\dot{\theta}^{(1)}(0). \tag{9}$$

Let us also require compliance with the periodicity conditions, i.e., let us seek periodic solutions of (6)

$$\theta^{(2)}(\pi/\omega) = \pm\theta^{(1)}(-\pi/\omega),\ \dot{\theta}^{(2)}(\pi/\omega) = \pm\dot{\theta}^{(1)}(-\pi/\omega). \tag{10}$$

Substituting solutions (7) and (8) into conditions (9) and (10), we obtain a system of linear homogeneous equations which has a nontrivial solution if its determinant equals zero, which results in the following relation

$$\cos\frac{\pi\alpha}{\omega}\cos\frac{\pi\beta}{\omega} - \frac{\alpha^2+\beta^2}{2\alpha\beta}\sin\frac{\pi\alpha}{\omega}\sin\frac{\pi\beta}{\omega} = \pm\operatorname{ch}\frac{2\pi\varepsilon}{\omega} \tag{11}$$

connecting the quantities Ra, D, δ, ε, and ω. Since $\mathrm{Ra} \sim \gamma$ and $D \sim \gamma^2$, then in order to analyze the influence of the temperature gradient on the convective stability explicitly, it is convenient to introduce the parameter $N = D/\mathrm{Ra}^2$ which is independent of γ.

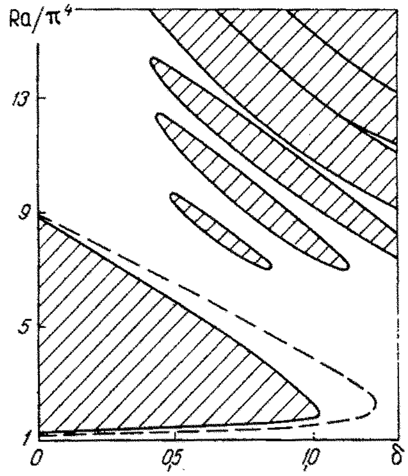
Fixing the quantities ε, δ, and N, we find the dependence of the critical value of the Rayleigh number Ra on the frequency numerically from (11). It turns out that the stability pattern depends essentially on the magnitude of the parameter N.

Let us first examine the case $N < N^* = \frac{1}{4}\pi^4(1+\delta^2)$. If there is no modulation, then the instability domain in the Rayleigh numbers has lower and upper bounds [7]

$$(1 - \sqrt{1 - 4\pi^4 N})/2N < \mathrm{Ra} < (1 + \sqrt{1 - 4\pi^4 N})/2N$$

and is independent of the damping parameter ε. In the presence of modulation ($\delta \neq 0$), parametric instability domains appear in addition to the fundamental instability strip. The stability and instability domains on the coordinate plane $(\mathrm{Ra}/\pi^4, 1/\omega)$ are shown in Fig. 1 for $\varepsilon = \sqrt{2}$, $\delta = 1$, $N = \frac{1}{10}\pi^4$. It is easy to see that at high frequencies the dependence of the critical value of the Rayleigh number of the frequency is monotonic in nature (there is no parametric instability). In this case, by expanding the left and right sides of (11) in power series in $1/\omega$ and δ, we obtain

$$\mathrm{Ra} = \pi^4 + \mathrm{Ra}^2 N(1+\delta^2) + \mathrm{Ra}^4 N^2 \delta^2/4\omega^2. \tag{12}$$



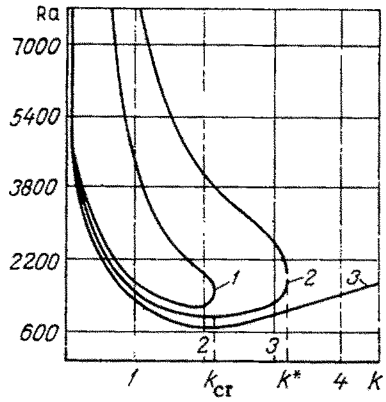


Fig. 3 Fig. 4

Fig. 3. Dependences of the critical value of the Rayleigh number on the modulation amplitude.

Fig. 4. Neutral curves: 1) $N = 1/400$; 2) $1/800$; 3) $1/10^4$.

The term dependent on the frequency in the right side drops out in the limit $\omega \to \infty$ and (12) goes over into the more simple expression

$$\mathrm{Ra} = \pi^4 + \mathrm{Ra}^2 \mathrm{N}(1 + \delta^2). \tag{13}$$

Solving (13) for Ra, we obtain the boundary of the fundamental instability band as $\omega \to \infty$

$$\frac{1 - \sqrt{1 - 4\pi^4 \mathrm{N}(1+\delta^2)}}{2\mathrm{N}(1+\delta^2)} \leqslant \mathrm{Ra} \leqslant \frac{1 + \sqrt{1 - 4\pi^4 \mathrm{N}(1+\delta^2)}}{2\mathrm{N}(1+\delta^2)}. \tag{14}$$

The boundaries of the fundamental instability domain, defined by (14), are shown in Figs. 1 and 3 by dashes.

The width of the fundamental instability band depends substantially on the parameters N, δ and vanishes completely for $[N > N^* = 1/4\pi^4(1+\delta^2)]$ (Fig. 2, $\varepsilon = \sqrt{2}$, $\delta = 1$, $N = 3/16\pi^4 (N^* = 1/8\pi^4)$). A stability domain (whose width grows with the increase in ε, δ, and N) exists above the fundamental instability domain and separates it from the parametric instability domains. These domains contract as ε increases. They shift to the higher Rayleigh numbers as N grows. The parametric instability domains include the broadest frequency bands for $\delta = 1$. As δ diminishes, they shift into the high frequency domain and vanish entirely for low modulation amplitudes. For high Rayleigh numbers $\mathrm{Ra} \gg 1/N$ and $\delta = 1$ the equations of the lines separating adjacent instability domains have the form

$$\frac{2\pi^4 \mathrm{Ra}\sqrt{\mathrm{N}}}{\omega} = j \; (j = 0,\ 1,\ 2\ \ldots).$$

In order to illustrate the dependence of the critical value of the Rayleigh number on the modulation amplitude, the dependence Ra(δ) is shown in Fig. 3 for $\omega = 1$.

Therefore, periodic modulation of a homogeneous magnetic field substantially influences the condition for the origination of convection. In this case the stability boundary is determined not only by the temperature gradient, the magnitude of the magnetic field, and the characteristics of the magnetizable fluid, but also depends in a complicated way on the modulation frequency and amplitude.

At high and low frequencies this dependence is monotonic in nature (response domains do not exist). In the intermediate case, resonance instability (stability) domains (see Fig. 3) appear with the increase in the amplitude.

5. Let us consider the convective stability of a horizontal layer of magnetizable fluid in a rotating field

$$H_{x0} = \cos\omega t,\ H_{y0} = \sin\omega t,\ H_{z0} = 0.$$

As has been shown above, this problem reduces to the investigation of solutions of (6). For arbitrary values of the parameters this equation can be solved only numerically, but in certain limit cases (at high frequencies, for example) it can be solved analytically.

Assuming the frequency "high," let us use the method of averaging [10], then by simple calculations we arrive at the stability boundary equation

$$\mathrm{Ra} = \frac{(k^2+n^2\pi^2)^3}{k^2} + \mathrm{Ra}^2\,\mathrm{N}\,\frac{k^2}{2\,(k^2+n^2\pi^2)} + \frac{k^6\,\mathrm{Ra}^4\,\mathrm{N}^2}{32\,(k^2+n^2\pi^2)^5\,\omega^2}\,. \tag{15}$$

Thus, the stability boundary has been found to the accuracy of the terms $1/\omega^2$, and Eq. (15) obtained is analyzed numerically only. Hence, by passing to the limit $\omega \to \infty$, we have a quadratic equation in Ra for the stability boundary, which we solve to obtain

$$\mathrm{Ra} = [1 \pm \sqrt{1-2\mathrm{N}(k^2+n^2\pi^2)}]\,(k^2+n^2\pi^2)/\mathrm{N}\,k^2. \tag{16}$$

It is seen that for $N > 1/(k^2 + n^2\pi^2)^2$ absolute stabilization of the horizontal layer of a magnetizing fluid by a homogeneous rotating magnetic field holds.

The family of neutral curves Ra(k) for the first mode (n = 1) is shown in Fig. 4. It is seen that the critical value of the Rayleigh number increases with the growth of N, while the critical wave number k_{cr} decreases.

Absolute stabilization sets in for $N = 1/2\pi^4$; if $0 < N < 1/2\pi^4$, then the wavenumbers of the perturbations causing the instability have the upper bound $k < k^*$.

Therefore, a high-frequency rotating magnetic field substantially raises the threshold stability of a magnetizable fluid layer heated from below up to total stabilization of the layer.

The possibility of the origination of macrorotations under the effect of a rotating magnetic field should be kept in mind in applying these results to a ferromagnetic fluid.

NOTATION

M and **H**, fluid magnetization and magnetic field intensity vectors; M and H, their magnitudes; T, temperature; v_z, vertical velocity component; Φ, magnetic field perturbation potential $H' = \nabla\Phi$; θ, temperature perturbation; ρ, density; χ, magnetic permeability; ν, coefficient of kinematic viscosity; $\varkappa$, coefficient of thermal diffusivity; γ, temperature gradient; g, acceleration of earth's gravitation; l, layer height; x, y, z, Cartesian coordinates; $k^2 = k_x^2 + k_y^2$, square of the wave number; $D = \mu_0(\partial M/\partial T)^2\gamma^2 l^4/\nu\varkappa\rho(1 + \chi)$, parameter characterizing the contribution of the magnetic field perturbations; $Ra = \beta g\gamma l^4/\nu\varkappa$, Rayleigh number; $Pr = \nu/\varkappa$, Prandtl number; $\beta = -1/\rho(\partial\rho/\partial T)$; $\varepsilon = (1+Pr)/\sqrt{Pr}$, damping parameter; $\Delta_1 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

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